Northern Beaches Secondary College

Manly Campus

2023 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. Which of the following is a solution to the equation $|\, e^{i\theta} - 1 \,| = 2$?

A. $\theta = \pi$

B. $\theta = \frac{\pi}{2}$

C. $\theta = 0$

D. $\theta = -\frac{\pi}{2}$

2. What is the contrapositive of the following statement?

If you're sad and you know it, then you will stomp your feet.

A. If you don't stomp your feet, then you're sad and you know it.

B. If you stomp your feet, then you're either not sad or you don't know it.

C. If you don't stomp your feet, then you're not sad and you don't know it.

D. If you don't stomp your feet, then you're either not sad or you don't know it.

3. The point (-1, a) lies on the line with vector equation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, where $\mu \in \mathbb{R}$.

Which of the following is the correct value of a ?

A. $-\frac{2}{3}$ B. $\frac{2}{3}$ C. $\frac{5}{4}$ D. 2

4. It is given that a, b, c and d are consecutive integers.

Which of the following statements may be false?

A. $abcd$ is divisible by 8

B. $abcd$ is divisible by 3

C. $a + b + c + d$ is divisible by 4

D. $a + b + c + d$ is divisible by 2

5. The polynomial $P(z)$ has real coefficients, and $P(2 - i) = 0$.

Which quadratic polynomial must be a factor of $P(z)$?

A. $z^2 + 4z + 5$

B. $z^2 - 4z + 5$

C. $z^2 + 4z + 3$

D. $z^2 - 4z + 3$

6. P, Q, R are three collinear points with position vectors $\underset{\sim}{\mathbf{p}}$, $\underset{\sim}{\mathbf{q}}$ and $\underset{\sim}{\mathbf{r}}$ respectively.

Q lies between P and R.

If $\left|\overrightarrow{QR}\right| = \frac{1}{2}\left|\overrightarrow{PQ}\right|$, then $\underset{\sim}{\mathbf{r}}$ is equal to

A. $\frac{3}{2}\underset{\sim}{\mathbf{p}} - \frac{1}{2}\underset{\sim}{\mathbf{q}}$

B. $\frac{3}{2}\underset{\sim}{\mathbf{q}} - \frac{1}{2}\underset{\sim}{\mathbf{p}}$

C. $\frac{1}{2}\underset{\sim}{\mathbf{p}} - \frac{3}{2}\underset{\sim}{\mathbf{q}}$

D. $\frac{1}{2}\underset{\sim}{\mathbf{p}} - \frac{3}{2}\underset{\sim}{\mathbf{q}}$

7. A particle P is moving in simple harmonic motion. Its maximum speed is $6\pi\ ms^{-1}$ and its displacement at this time is 4 metres.

After 10 seconds it has reached its maximum speed again.

What is a possible equation for the displacement of P?

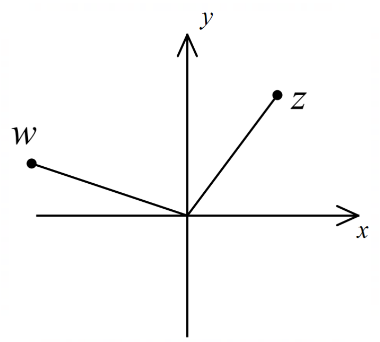
A. $x = 60\sin\left(\frac{\pi t}{10}\right) + 4$

B. $x = 60\sin\left(\frac{\pi t}{20}\right) + 4$

C. $x = 60\sin(10t) + 4$

D. $x = 60\sin(20t) + 4$

8. The Argand diagram shows the complex numbers z and w, where z lies in the first quadrant and w lies in the second quadrant.



Which complex number could lie in the third quadrant?

A. $-w$

B. $2iz$

C. $\overline{z}$

D. $w - z$

9. Consider the complex numbers $z_1 = 1 - i$ and $z_2 = 1 + \sqrt{3}\, i$.

What is the smallest positive value of n such that $\left(\dfrac{z_2}{z_1}\right)^n$ is purely imaginary?

A. 2

B. 4

C. 6

D. 8

10. Without evaluating the integrals, which one of the following integrals is greater than zero?

A. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x}{2 + \cos x}\, dx$

B. $\displaystyle\int_{-\pi}^{\pi} x^3 \sin x \, dx$

C. $\displaystyle\int_{-1}^{1} (e^{-x^2} - 1)\, dx$

D. $\displaystyle\int_{-2}^{2} \tan^{-1}(x^3)\, dx$

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions is Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new Writing Booklet.

a) Write the negation of the statement *P*: "I am both rich and happy." **2**

b) Suppose $p \in \mathbb{R}$ satisfies $7^p = 2$. Prove that *p* is irrational. **2**

c) It is given that the point *R* is (2, 1, -1), $\overrightarrow{RS} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix}$ and $\overrightarrow{RT} = 3\overrightarrow{RS}$.

Find the coordinates of *T*. **2**

d) Simplify $\left(e^{-\frac{i\pi}{3}}\right)^2 \left(1 + i\sqrt{3}\right)^3$, writing your answer in the form $x + iy$ where $x, y \in \mathbb{R}$. **2**

e) On an Argand diagram, sketch the region satisfied by both inequalities:

$|z + 1| \leq |z - i|$ and $Im(z) < 2$ **3**

f)

(i) Show that $\dfrac{1}{x(1 + x^2)} = \dfrac{1}{x} - \dfrac{x}{1 + x^2}$ **1**

(ii) Hence find $\displaystyle\int \frac{1}{x^2} \tan^{-1} x \, dx$ **3**

Question 12 (15 marks) Start a new Writing Booklet.

a) Calculate $\int_1^3 x\sqrt{\frac{x-1}{2}}\,dx$ **3**

b) Using the substitution $x = 2 + 2\cos^2\theta$, calculate the value of $\int_2^3 \sqrt{\frac{x-2}{4-x}}\,dx$ **4**

c)

(i) Find the square roots of $-3 - 4i$ **2**

(ii) Hence or otherwise, solve the equation $z^2 - 3z + (3 + i) = 0$ **2**

d) Consider the line $l = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ and the points $A = (1, 2, 3)$ and $B = (0, 3, 8)$.

(i) Find the projection of the vector $\overrightarrow{AB}$ on the line l. **2**

(ii) Hence or otherwise, find the shortest distance from point B to the line l. **2**

Question 13 (15 marks) Start a new Writing Booklet.

a) Prove that the difference between the two-digit numbers “AB” and “BA” is always divisible by 9. **2**

b) Using the substitution $t = \tan\frac{x}{2}$ or otherwise, evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2}{\sin x + 1}\, dx$. Give your answer in the form $a + b\sqrt{3}$ where $a, b \in \mathbb{R}$ **3**

c) The line l_1 has equation $\underset{\sim}{\mathbf{r}} = 2\underset{\sim}{\mathbf{i}} + 3\underset{\sim}{\mathbf{k}} + \lambda\left(\underset{\sim}{\mathbf{i}} - \underset{\sim}{\mathbf{j}} + 2\underset{\sim}{\mathbf{k}}\right)$ where λ is a parameter.

The line l_2 has the equation $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$.

(i) Find the point of intersection of l_1 and l_2 **3**

(ii) To the nearest minute, find the acute angle between l_1 and l_2 . **2**

d)

(i) Given that $a > 0$, $b > 0$, prove that $a + b \geq 2\sqrt{ab}$ **1**

(ii) Hence show that $\sec^2 x \geq 2\tan x$ **1**

(iii) Prove that $\sec^{2n} x + \mathrm{cosec}^{2n} x \geq 2^{n+1}$ for all integers $n \geq 0$ and for all $x \in \left(0, \frac{\pi}{2}\right)$ **3**

Question 14 (15 marks) Start a new Writing Booklet.

a) Let ω be a complex fifth root of unity, that is, $\omega^5 = 1$ and $\omega \neq 1$.

(i) Show that $\omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0$ **1**

(ii) Prove that one root of the equation $z^4 + 5z^2 + 5 = 0$ is $z = \omega - \omega^4$ **3**

b) Find the Cartesian equation of the sphere with centre $\underset{\sim}{\mathbf{c}} = 2\underset{\sim}{\mathbf{i}} - 3\underset{\sim}{\mathbf{j}} + 4\underset{\sim}{\mathbf{k}}$ which passes through the point (1, -1, 2). **2**

c) (i) Show that $\frac{d}{dx}(\sec 2x) = 2 \tan 2x \sec 2x$ **2**

(ii) Find $\int \tan^3 2x \sec 2x \, dx$ **2**

d) Let α and β be conjugate complex numbers such that $|\alpha - \beta| = 4\sqrt{3}$ and $\frac{\alpha}{\beta^2}$ is real.

(i) Show that α^3 is real **2**

(ii) Calculate $|\alpha|$ **3**

Question 15 (15 marks) Start a new Writing Booklet.

a) A particle is moving horizontally at time t seconds such that $v^2 = x(6 - x)$, where v is the velocity in ms^{-1} and x is the displacement in metres.

(i) Prove that the particle is moving in simple harmonic motion. **2**

(ii) Find the centre and amplitude of the motion. **2**

(iii) Find the period of the motion. **1**

(iv) Find the maximum speed. **1**

(v) At $t = 0$, $v = 3$ and $x = 3$.
Find the displacement x as a function of time t. **3**

b) The polynomial $P(z) = z^4 - 10z^3 + cz^2 + dz + 169$ has two zeroes $\alpha = a + ib$ and $\beta = b + ia$, where $a, b, c, d \in \mathbb{R}$ and $a \neq b$.

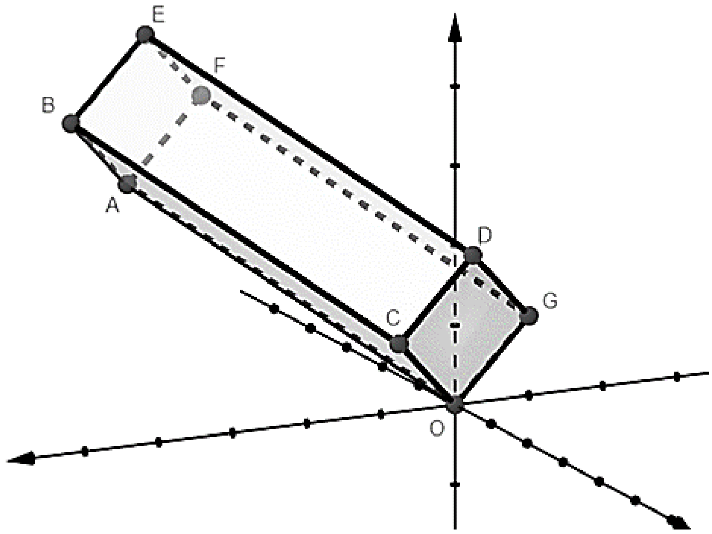
(i) Express the other two zeroes of $P(z)$ in terms of a and b, justifying your answer. **1**

(ii) Find the values of a and b, given that they are both integers. **2**

c) Use mathematical induction to show that $\left(2 - \dfrac{1}{n}\right)^n > n$ for all integers $n \geq 2$ **3**

Question 16 (15 marks) Start a new Writing Booklet.

a) The diagram shows a rectangular prism.



Let $\overrightarrow{OA} = 3\underset{\sim}{\mathbf{i}} - 12\underset{\sim}{\mathbf{j}} + 3\underset{\sim}{\mathbf{k}}$, $\overrightarrow{OC} = 2\underset{\sim}{\mathbf{i}} + \underset{\sim}{\mathbf{j}} + a\underset{\sim}{\mathbf{k}}$, and $\overrightarrow{OG} = -2\underset{\sim}{\mathbf{i}} + y\underset{\sim}{\mathbf{j}} + z\underset{\sim}{\mathbf{k}}$.

(i) Show that $a = 2$ **1**

(ii) Hence or otherwise, show that $y = 0$ and $z = 2$. **3**

(iii) Calculate how high point E is above the x-y plane. **2**

b) Find $\displaystyle\int \frac{x^2 - 7x + 4}{(x+1)(x-1)^2}\,dx$ **4**

c) (i) Let $\underset{\sim}{\mathbf{a}} = a_1\underset{\sim}{\mathbf{i}} + a_2\underset{\sim}{\mathbf{j}} + a_3\underset{\sim}{\mathbf{k}}$ and $\underset{\sim}{\mathbf{b}} = b_1\underset{\sim}{\mathbf{i}} + b_2\underset{\sim}{\mathbf{j}} + b_3\underset{\sim}{\mathbf{k}}$.

By considering $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}}$, show that:

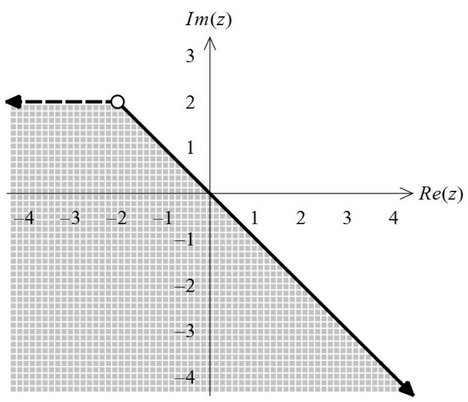
$$\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right) \geq (a_1 b_1 + a_2 b_2 + a_3 b_3)^2$$ **2**

(ii) Given that x, y and z are non-negative and that $x + y + z = 3$, find the minimum possible value of $x^2 + 4y^2 + z^2$. **3**

End of paper

2023 Year 12 Mathematics Extension 2 Trial solutions		
Q	**Solution**	**Mark allocation**
1	$\lvert\cos\theta + i\sin\theta - 1\rvert = 2$ $\sqrt{(\cos\theta - 1)^2 + sin^2\theta} = 2$ $\sqrt{2 - 2\cos\theta} = 2$ $2 - 2\cos\theta = 4$ $\cos\theta = -1$ $\therefore \theta = \pi$	A ***29 (of 38) students answered correctly***
2	Statement is of the form $A \cap B \Rightarrow C$ Contrapositive of this statement is: $\overline{C} \Rightarrow \overline{A} \bigcup \overline{B}$	D ***36 students answered correctly***
3	$x = 2 - 3\mu$, $y = -2 + 4\mu$ When forming a Cartesian equation, we have $y = -2 + 4\left(\frac{2-x}{3}\right)$ When $x = -1$, $y = 2$ Therefore $a = 2$	D ***38 students answered correctly!***
4	Option C: A counter-example is 1+2+3+4=10 which is not divisible by 4. Option A: In four consecutive integers, one integer is divisible by 2 while another integer is divisible by 4. So the product is divisible by 8. Option B: In four consecutive integers, one of them is divisible by 3. So the product is divisible by 3. Option D: In four consecutive integers, 2 of them are even, and two of them are odd, so the sum is even.	C ***31 students answered correctly***
5	By the conjugate root theorem, both $2 + i$ and $2 - i$ are factors, $\therefore\ [z - (2 + i)][z - (2 - i)]$ $= (z - 2 - i)(z - 2 + i)$ $= (z - 2)^2 - i^2$ $= z^2 - 4z + 4 + 1$ $= z^2 - 4z + 5$ is a factor	B ***35 students answered correctly***
6	$\underset{\sim}{\mathbf{r}} = \overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PQ} + \overrightarrow{QR}$ $= \overrightarrow{OP} + \overrightarrow{PQ} + \frac{1}{2}\overrightarrow{PQ}$ $= \overrightarrow{OP} + \frac{3}{2}\overrightarrow{PQ}$ $= \underset{\sim}{\mathbf{p}} + \frac{3}{2}\left(\underset{\sim}{\mathbf{q}} - \underset{\sim}{\mathbf{p}}\right)$ $= \frac{3}{2}\underset{\sim}{\mathbf{q}} - \frac{1}{2}\underset{\sim}{\mathbf{p}}$	B ***31 students answered correctly***
7	$x = A\sin(nt) + k$ $\dot{x} = An\cos(nt)$ Maximum speed = An since $-1 \leq \cos(nt) \leq 1$ $An = 6\pi$	A ***34 students answered correctly***

	Maximum speed occurs again after half a period, so the period is 20 seconds $\frac{20\pi}{n} = 20 \Rightarrow n = \frac{\pi}{10}$ $\therefore A \times \frac{\pi}{10} = 6\pi$ $A = 60$ $t = 0, x = 4 \Rightarrow k = 4$ $\therefore x = 60\sin\left(\frac{\pi t}{10}\right) + 4$	
8	Option A: -*w* lies in the 4th quadrant (rotation of *w* by 180 degrees about the origin) Option B: 2*iz* lies in the 2nd quadrants (rotation of *z* by 90 degrees and double the modulus) Option C: $\bar{z}$ lies in the 4th quadrant (reflection of *z* about *x*-axis) Option D: Adding the vectors *w* and -*z* could take us to the 3rd quadrant, depending on the lengths of these vectors	D ***35 students answered correctly***
9	$\left(\frac{z_2}{z_1}\right)^n = \left(\frac{1+\sqrt{3}\,i}{1-i}\right)^n = \left(\frac{2e^{\frac{i\pi}{3}}}{\sqrt{2}\,e^{-\frac{i\pi}{4}}}\right)^n = \left(\sqrt{2}\,e^{\frac{7\pi i}{12}}\right)^n = \left(\sqrt{2}\right)^n e^{\frac{7\pi n i}{12}}$ For this to be purely imaginary, we require the argument to be a multiple of $\frac{\pi}{2}$ Hence the smallest positive value of *n* is 6.	C ***36 students answered correctly***
10	Option A: $f(-x) = \frac{-x}{2+\cos(-x)} = -\frac{x}{2+\cos x} = f(x)$ Hence the function is odd, and the integral $\int_{-a}^{a}$ is 0. Option D: same reasoning as A Option C: the graph of $y = f(x)$ is below the *x*-axis for all real *x*, so the integral is less than 0. Option B: $x^3\sin x$ is non-negative for [-π, π] , hence the integral is positive.	B ***19 students answered correctly***
11a	Negation: "I am not rich *or* I am not happy."	**2 Marks:** Correct answer **1 Mark:** "I am not rich **and** I am not happy" OR <u>One</u> of the statements: "I am not rich", "I am not happy" ***23 students got full marks***

11b	Assume that there is a rational number $p=\frac{a}{b}$, where $a,b\in\mathbb{Z}$ such that $7^p=2$ That is: $7^{a/b}=2$ $\Rightarrow 7^a=2^b$ For integral a,b, the left side of this equation is odd whilst the right side is even. This is a contradiction; hence p must be irrational.	**2 Marks:** Correct answer. **1 Mark:** Obtains the expression $7^{a/b}=2$. ***21 students got full marks***
11c	$\overrightarrow{OT}=\overrightarrow{OR}+\overrightarrow{RT}$ $=\overrightarrow{OR}+3\overrightarrow{RS}$ $=\begin{pmatrix}2\\1\\-1\end{pmatrix}+3\begin{pmatrix}-4\\-1\\2\end{pmatrix}$ $=\begin{pmatrix}-10\\-2\\5\end{pmatrix}$	**2 Marks:** Correct answer. **1 Mark:** Attempts to write a vector equation to find $\overrightarrow{OT}$ ***31 students got full marks***
11d	$\left(e^{-i\pi/3}\right)^2\left(1+i\sqrt{3}\right)^3=e^{-i2\pi/3}\times\left(2e^{i\pi/3}\right)^3$ $=e^{-i2\pi/3}\times 8e^{i\pi}$ $=8e^{i\pi/3}$ $=8\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$ $=4+4\sqrt{3}i$	**2 Marks:** Correct answer. **1 Mark:** Converts $1+i\sqrt{3}$ to exponential form or converts both terms to *cis* form. ***23 students got full marks***
11e	Let $z=x+iy$ $\lvert x+1+iy\rvert\le\lvert x+i(y-1)\rvert$ $(x+1)^2+y^2\le x^2+(y-1)^2$ $y\le -x$ 	**3 marks:** Correct answer. **2 Marks:** Correct sketch of each region but does not show correct intersection. **1 Mark:** Correct sketch of one region, OR Correct algebraic representation of one of $\lvert z+1\rvert$ or $\lvert z-i\rvert$. ***12 students got full marks***

11fi	$\frac{1}{x} - \frac{x}{1+x^2} = \frac{(1+x^2)-x^2}{x(1+x^2)}$ $= \frac{1+x^2-x^2}{x(1+x^2)}$ $= \frac{1}{x(1+x^2)}$	**1 Mark:** Correct answer.
11fii	Let $u = \tan^{-1}x \Rightarrow du = \frac{1}{1+x^2}$ $v' = \frac{1}{x^2} \Rightarrow v = -\frac{1}{x}$ $\therefore \int \frac{1}{x^2}\tan^{-1}x\,dx$ $= -\frac{1}{x}\tan^{-1}x - \int -\frac{1}{x} \times \frac{1}{1+x^2}\,dx$ $= -\frac{1}{x}\tan^{-1}x + \int \frac{1}{x(1+x^2)}\,dx$ $= -\frac{1}{x}\tan^{-1}x + \int \left[\frac{1}{x} - \frac{x}{1+x^2}\right] dx$ $= -\frac{1}{x}\tan^{-1}x + \int \frac{1}{x}\,dx - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$ $= -\frac{1}{x}\tan^{-1}x + \ln\lvert x\rvert - \frac{1}{2}\ln(1+x^2) + C$	**3 Marks:** Correct answer. **2 Marks:** Correct use of IBP to obtain the expression in line © **1 Mark:** Obtains correct expressions for *du* and *v*. ***30 students got full marks for Q11f***
12a	Let $u^2 = \frac{x-1}{2} \Rightarrow x = 1+2u^2$ Then $dx = 4u\,du$ and $x=1 \Rightarrow u=0, x=3 \Rightarrow u=1$ $\int_1^3 x\sqrt{\frac{x-1}{2}}dx = \int_0^1 (1+2u^2)u \times 4u\,du$ $= 4\int_0^1 (u^2+2u^4)\,du$ $= 4\left[\frac{u^3}{3} + \frac{2u^5}{5}\right]_0^1$ $= 4\left(\frac{1}{3} + \frac{2}{5} - 0\right)$ $= \frac{44}{15}$	**3 Marks:** Correct answer. **2 Marks:** Makes a suitable substitution and correctly transforms the integrand. **1 Mark:** Identifies a suitable substitution and correctly transforms *dx*. ***24 students got full marks.*** ***Most students made a suitable substitution. The most common mistakes were algebraic.***
12b	$x = 2+2\cos^2\theta \Rightarrow dx = -4\cos\theta\sin\theta\,d\theta$ $x = 2 \Rightarrow \theta = \frac{\pi}{2}$ $x = 3 \Rightarrow \theta = \frac{\pi}{4}$	**4 Marks:** Correct answer. **3 Marks:** Correctly transforms given integral to the

	$\int_2^3 \sqrt{\frac{x-2}{4-x}}dx = \int_{\pi/2}^{\pi/4} \sqrt{\frac{2\cos^2\theta}{2-2\cos^2\theta}}(-4\cos\theta\sin\theta)\,d\theta$ $= \int_{\pi/4}^{\pi/2} \sqrt{\frac{\cos^2\theta}{\sin^2\theta}} 4\cos\theta\sin\theta\,d\theta$ $= 4\int_{\pi/4}^{\pi/2} \frac{\cos\theta}{\sin\theta}\cos\theta\sin\theta\,d\theta$ $= 4\int_{\pi/4}^{\pi/2} \cos^2\theta\,d\theta$ $= \int_{\pi/4}^{\pi/2} (2\cos 2\theta + 2)\,d\theta$ $= [\sin 2\theta + 2\theta]_{\pi/4}^{\pi/2}$ $= \sin\pi + \pi - \left(\sin\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= \frac{\pi}{2} - 1$	equivalent expression in terms of $\cos^2\theta$ **2 Marks:** Correctly transforms differential and limits. **1 Mark:** Correctly transforms integrand **or** limits **or** differential. ***12 students got full marks.*** ***The most common mistakes were algebraic (trig identities, trig exact values, minus signs, coefficients).***
12ci	Solve for x and y, where $x, y \in \mathbb{R}$, such that $(x+iy)^2 = -3-4i$ $x^2 - y^2 + 2xyi = -3-4i$ Equating real and imaginary parts gives $x^2 - y^2 = -3$ and $xy = -2$ Solving the equations simultaneously, $x^2 - \left(\frac{-2}{x}\right)^2 = -3$ $x^4 + 3x^2 - 4 = 0$ $(x^2+4)(x^2-1) = 0$ Since $x \in \mathbb{R}$, $x^2 = 1$. $\therefore x = \pm 1, y = \mp 2$ The square roots are $1-2i$ and $-1+2i$	**2 Marks:** Correct answer. **1 Mark:** Forms correct expressions for $x^2 - y^2$ and xy
12cii	$z = \frac{3\pm\sqrt{9-4(3+i)}}{2} = \frac{3\pm\sqrt{-3-4i}}{2}$ Using part (i), the required solutions are $z = \frac{3+1-2i}{2}, \frac{3-1+2i}{2}$ $\therefore z = 2-i, 1+i$	**2 Marks:** Correct answer {consistent with results from part (i)} **1 Mark:** Correct substitution into quadratic formula. ***22 students got full marks for Q12c. Mostly done well. This is a stock-standard procedure in Ext 2.***
12di	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 0-1 \\ 3-2 \\ 8-3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$	**2 Marks:** Correct answer.

	$\therefore proj_l\overrightarrow{AB} = proj_{\underline{b}}\overrightarrow{AB}$, where $\underline{b} = \begin{pmatrix}1\\-1\\4\end{pmatrix}$ is the direction vector of the line. $$\therefore proj_l\overrightarrow{AB} = \frac{\overrightarrow{AB}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\underline{b} = \frac{-1-1+20}{1+1+16}\begin{pmatrix}1\\-1\\4\end{pmatrix} = \begin{pmatrix}1\\-1\\4\end{pmatrix}$$	**1 Mark:** Finds the vector $\overrightarrow{AB}$ ***Note that "l" is a line, not a vector. Two vectors are required in order to use the projection formula. A diagram may be helpful before you begin.*** ***A surprising number of students didn't write the projection formula correctly (from Ext 1).***
12dii	$\perp proj_l\overrightarrow{AB} = \overrightarrow{AB} - proj_l\overrightarrow{AB} = \begin{pmatrix}-1\\1\\5\end{pmatrix} - \begin{pmatrix}1\\-1\\4\end{pmatrix} = \begin{pmatrix}-2\\2\\1\end{pmatrix}$ $\therefore shortest\ distance = \left\|\perp proj_l\overrightarrow{AB}\right\| = \sqrt{(-2)^2+2^2+1^2} = 3$	**2 Marks:** Correct answer. **1 Mark:** Finds the perpendicular vector. ***8 students got full marks for Q12d.***
13a	We can write $"AB" = 10A + B$ And $"BA" = 10B + A$ $\therefore "AB" - "BA" = 10A + B - (10A + B)$ $= 10A + B - 10B - A$ $= 9A - 9B$ $= 9(A - B)$	**2 Marks:** Correct answer **1 Mark:** Correct expanded expression for either "*AB*" or "*BA*" ***32 students got full marks***
13b	Let $t = \tan\frac{x}{2}$ => $dt = \frac{1}{2}\sec^2\frac{x}{2}dx = \frac{1}{2}(1+t^2)dx$ => $dx = \frac{2}{1+t^2}dt$ When $x = \frac{\pi}{3}, t = \frac{1}{\sqrt{3}}$ $x = \frac{\pi}{2}, t = 1$ $$\therefore I = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2}{\sin x + 1}dx = \int_{\frac{1}{\sqrt{3}}}^{1} \frac{2}{\frac{2t}{1+t^2}+1} \times \frac{2}{1+t^2}dt = \int_{\frac{1}{\sqrt{3}}}^{1} \frac{4}{2t+1+t^2}dt$$ $$I = \int_{\frac{1}{\sqrt{3}}}^{1} \frac{4}{(t+1)^2}dt = \left[-\frac{4}{t+1}\right]_{\frac{1}{\sqrt{3}}}^{1} = -\frac{4}{1+1} - \left(-\frac{4}{\frac{1}{\sqrt{3}}+1}\right)$$ $$I = -2 + \frac{4\sqrt{3}}{1+\sqrt{3}} = -2 + \frac{4\sqrt{3}(1-\sqrt{3})}{1-3} = -2 - 2\sqrt{3}(1-\sqrt{3})$$ $\therefore I = 4 - 2\sqrt{3}$	**3 Marks:** Correct answer. **2 Marks:** Correct substitution of *t* formula, including correct transformations of integrand, differential and limits. **1 Mark:** Correct substitution of *t* formula but incorrect/incomplete transformation of integrand, differential or limits.

		19 students got full marks
13ci	$\underset{\sim}{\mathbf{r}} = 2\underset{\sim}{\mathbf{i}} + 3\underset{\sim}{\mathbf{k}} + \lambda\left(\underset{\sim}{\mathbf{i}} - \underset{\sim}{\mathbf{j}} + 2\underset{\sim}{\mathbf{k}}\right)$ $= 2\underset{\sim}{\mathbf{i}} + \lambda\underset{\sim}{\mathbf{i}} - \lambda\underset{\sim}{\mathbf{j}} + 2\lambda\underset{\sim}{\mathbf{k}} + 3\underset{\sim}{\mathbf{k}}$ $= (2 + \lambda)\underset{\sim}{\mathbf{i}} - \lambda\underset{\sim}{\mathbf{j}} + (2\lambda + 3)\underset{\sim}{\mathbf{k}}$ $l_1 : \underset{\sim}{\mathbf{r}} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-4}{3} = \mu$ $\frac{x-2}{2} = \mu \Rightarrow x = 2 + 2\mu$ $\frac{y+1}{-1} = \mu \Rightarrow y = -1 - \mu$ $\frac{z-4}{3} = \mu \Rightarrow z = 4 + 3\mu$ $l_2 : \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}\mu$ $\therefore \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}\mu$ $\Rightarrow \quad -\lambda = -1 - \mu$ $\lambda = 1 + \mu$ $2 + \lambda = 2 + 2\mu$ $2 + 1 + \mu = 2 + 2\mu$ $\mu = 1$ $\lambda = 2$ $3 + 2\lambda = 3 + 2 \times 2 = 7$ $4 + 3\mu = 4 + 3 \times 1 = 7$ $\therefore$ the lines intersect at $\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$	**3 Marks:** Correct answer **2 Marks:** Finds correct equations for l_1 and l_2 **1 Mark:** Finds correct equation for l_1 OR l_2.

13cii	$\cos\theta = \dfrac{\begin{pmatrix}2\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\2\end{pmatrix}}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$ $= \dfrac{2+1+6}{\sqrt{14}\sqrt{6}}$ $= \dfrac{9}{\sqrt{84}}$ $\therefore \theta = 10^\circ 53`36.22`` \approx 10^\circ 54`$ to nearest minute	**2 Marks:** Correct answer. **1 Mark:** Finds a value for θ consistent with an incorrect value of cosθ. ***4 students got full marks for Q13c***
13di	$\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$ $a - 2\sqrt{ab} + b \geq 0$ $a + b \geq 2\sqrt{ab}$	**1 Mark:** Correct answer.
13dii	$\sec^2 x = \tan^2 x + 1 \geq 2\sqrt{\tan^2 x \times 1} = 2\tan x$	**1 Mark:** Correct answer.
13diii	Similar to (ii), $\text{cosec}^2 x = \cot^2 x + 1 \geq 2\cot x$ Note that for $x \in \left(0, \dfrac{\pi}{2}\right)$, $\tan x > 0$ and $\cot x > 0$ Then $\sec^{2n} x = \left(\sec^2 x\right)^n \geq \left(2\tan x\right)^n = 2^n \tan^n x$ and $\text{cosec}^{2n} x = \left(\text{cosec}^2 x\right)^n \geq \left(2\cot x\right)^n = 2^n \cot^n x$ So $\sec^{2n} x + \text{cosec}^{2n} x \geq 2^n \tan^n x + 2^n \cot^n x$ $= 2^n\left(\tan^n x + \cot^n x\right)$ $\geq 2^n\left(2\sqrt{\tan^n x \times \cot^n x}\right)$ using part (a) $= 2^{n+1}$ since $\cot x = \dfrac{1}{\tan x}$	**3 Marks:** Correct answer. **2 Marks:** Forms expressions for $\sec^{2n}x$ and $\text{cosec}^{2n}x$ in terms of $\tan x$ and $\cot x$. **1 Mark:** Establishes $\text{cosec}^2x \geq 2\cot x$. ***5 students got full marks for Q13d***
14ai	$\omega^5 - 1 = 0$ $(\omega-1)\left(\omega^4+\omega^3+\omega^2+\omega+1\right) = 0$ Since $\omega \neq 1$, $\omega^4+\omega^3+\omega^2+\omega+1 = 0$ **OR** The roots of $\omega^5 - 1 = 0$ are $1, \omega, \omega^2, \omega^3, \omega^4$. Consider the sum of the roots: $1 + \omega + \omega^2 + \omega^3 + \omega^4 = -\dfrac{b}{a} = -\dfrac{0}{1} = 0$	**1 Mark:** Correct answer. ***This is a stock-standard proof in Ext 2.***

14aii	If $z=\omega-\omega^4$ $z^2=\left(\omega-\omega^4\right)^2$ $=\omega^2-2\omega^5+\omega^8$ $=\omega^2-2+\omega^3$, since $\omega^5=1$ $z^4=\left(\omega-\omega^4\right)^4$ $=\omega^4-4\omega^3\left(\omega^4\right)^1+6\omega^2\left(\omega^4\right)^2-4\omega\left(\omega^4\right)^3+\left(\omega^4\right)^4$ $=\omega^4-4\omega^7+6\omega^{10}-4\omega^{13}+\omega^{16}$ $=\omega^4-4\omega^2+6-4\omega^3+\omega$, since $\omega^5=1$ Hence $z^4+5z^2+5=\omega^4-4\omega^2+6-4\omega^3+\omega+5\left(\omega^2-2+\omega^3\right)+5$ $=\omega^4+\omega^3+\omega^2+\omega+1$ $=0$, using (i) So $z=\omega-\omega^4$ is a solution of $z^4+5z^2+5=0$	**3 Marks:** Correct answer. **2 Marks:** Correct **simplified** expressions for z^2 and z^4. **1 Mark:** Correct **unsimplified** expressions for z^2 **and** z^4 OR Correct simplified expression for z^2 **or** z^4. ***Students who wrote out P(w-w⁴) and expanded were able to gain at least 1 mark. 12 students got full marks for Q14a***
14b	$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$ $\begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix}=\begin{pmatrix}2\\-3\\4\end{pmatrix}$ $r^2=(1-2)^2+(-1-3)^2+(2-4)^2=9$ $\therefore\quad (x-2)+(y+3)+(z-4)^2=9$	**2 Marks:** Correct answer. **1 Mark:** Obtains correct LHS or RHS of equation of sphere. ***28 students answered correctly. Most common mistake was not writing in Cartesian form.***
14ci	$\frac{d}{dx}(\sec 2x)=\frac{d}{dx}(\cos 2x)^{-1}$ $=-1(\cos 2x)^{-2}\times -2\sin 2x$ $=\frac{2\sin 2x}{(\cos 2x)^2}$ $=2\times\frac{\sin 2x}{\cos 2x}\times\frac{1}{\cos 2x}$ $=2\tan 2x\sec 2x$	**2 Marks:** Correct answer. **1 mark:** Attempts to use the chain rule to differentiate $(\cos 2x)^{-2}$ ***Note: beware of 1 or 2-mark "prove" questions. Since the last line is given, you must take care to show all steps of working.***

<table>
<tr><td>14cii</td><td>

$I = \int \tan^3 2x \sec 2x \, dx$
$= \int \tan 2x(\tan^2 2x) \sec 2x \, dx$
$= \int \tan 2x (\sec^2 2x - 1)\sec 2x \, dx$
$= \frac{1}{2} \int (\sec^2 2x - 1) \, 2 \tan 2x \sec 2x \, dx$
$= \frac{1}{2} \int \sec^2 2x \, (2 \tan 2x \sec 2x) dx - \frac{1}{2} \int 2\tan 2x \sec 2x \, dx$
$= \frac{1}{2}\left(\frac{1}{3}\sec^3 2x\right) - \frac{1}{2}\sec 2x \, dx$
$= \frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + C$

</td><td>

2 Marks: Correct answer

1 Mark: Correct progress to third line of working, i.e. splits $\tan^3 2x$ into $\tan 2x*(\sec^2 2x-1)$

Most students overcomplicated things with integration by parts, or did not see how to apply (i).

2 students got full marks for Q14c.

</td></tr>
<tr><td>14di</td><td>

Since $\beta = \overline{\alpha}$, and $\alpha\overline{\alpha} = |\alpha|^2$
$\alpha^3 = \frac{\alpha}{\beta^2} \times (\alpha\beta)^2 = \frac{\alpha}{\beta^2} \times (\alpha\overline{\alpha})^2 = \frac{\alpha}{\beta^2} \times |\alpha|^4$
Then α^3 is real since $\frac{\alpha}{\beta^2}$ and $|\alpha|$ are real

<u>Alternative method:</u>

$\alpha = r\text{cis}\theta$

$\beta = \overline{\alpha} = r\text{cis}(-\theta)$

$\frac{\alpha}{\beta^2} = \frac{r\text{cis}\theta}{(r\text{cis}(-\theta))^2}$
$= \frac{r\text{cis}\theta}{r^2\text{cis}(-2\theta)}$
$= \frac{1}{r}\text{cis}(3\theta) \in \mathbb{R}$
$\therefore \text{cis } 3\theta \in \mathbb{R}$
$\therefore \alpha^3 = r^3 \text{ cis } 3\theta \in \mathbb{R}$

</td><td>

2 Marks: Correct answer.

1 Mark: Transforms α^3 to $\alpha/\beta^2 \times (\alpha\beta)^2$.

This part challenged most students. The easiest method involves the use of conjugate properties.

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<tr><td>14dii</td><td>

Let $\alpha = x + iy$ where $x, y \in \mathbb{R} \Rightarrow \beta = x - iy$
$|\alpha - \beta| = |2iy| = 2y = 4\sqrt{3} \therefore y = 2\sqrt{3}$

$\alpha^3 = x^3 - 3xy^2 + y(3x^2 - y^2)i$
α^3 is real, hence $y(3x^2 - y^2) = 0$
Since $y = 2\sqrt{3}$, $3x^2 - y^2 = 0 \Rightarrow 3x^2 = 12$
$\therefore x^2 = 4$
$\Rightarrow |\alpha| = \sqrt{x^2 + y^2} = \sqrt{4+12} = 4$

</td><td>

3 Marks: Correct answer.

2 Marks: Finds $y=\text{Im}(\alpha)$ **and** forms an expression for α^3 in terms of x and y.

1 Mark: Obtains $y = 2\sqrt{3}$.

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</table>

		4 students got full marks for Q14d
15ai	$v^2 = x(6 - x) = 6x - x^2$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(3x - \frac{1}{2}x^2\right) = 3 - x$ $\therefore \quad \ddot{x} = -1(x - 3)$ $\ddot{x}$ is of the form $-n^2(x - c)$ with $n = 1$ $\therefore$ *SHM*	**2 Marks:** Correct answer **1 Mark:** Obtains a correct derivative
15aii	From $\ddot{x} = -(x - 3)$, the centre is $x = 3$ $v = 0$ at amplitude position $x(6 - x) = 0 \Rightarrow x = 0, 6$ $\therefore$ amplitude = $3 - 0$ or $6 - 3 = 3$	**2 Marks:** Correct values for both centre and amplitude **1 Mark:** One correct value
15aiii	Period: $T = \frac{2\pi}{n} = \frac{2\pi}{1} = 2\pi$	**1 Mark:** Correct answer
15aiv	Maximum speed occurs at centre, $x = 3$ $v^2 = 3(6 - 3) = 9 \Rightarrow \lvert v\rvert_{max} = 3\ ms^{-1}$	**1 Mark:** Correct answer
15av	$t = 0, x = 3, v = 3$ Starts at centre of motion $\Rightarrow \quad x = A\sin(nt + \alpha) + c$ $n = 1, c = 3, A = 3$ $\therefore \quad x = 3\sin(t + \alpha) + 3$ $t = 0, x = 3 \Rightarrow 3\sin\alpha + 3 = 3$ $\sin\alpha = 0$ $\alpha = 0$ $\therefore \quad x = 3\sin t + 3$ OR $x = A\cos(nt + \alpha) + c$ $A = 3, n = 1, c = 3$ $x = 3\cos(t + \alpha) + 3$ $t = 0, x = 3 \Rightarrow 3 = 3\cos\alpha + 3$ $\Rightarrow \cos\alpha = 0$ $\alpha = \frac{\pi}{2}$ $\therefore \quad x = 3\cos\left(t + \frac{\pi}{2}\right) + 3$ OR	**3 Marks:** Correct answer **2 Marks:** Finds the correct value for α. **1 Mark:** Attempts to use a correct trigonometric expression for x. OR **3 Marks:** Correct answer **2 Marks:** Forms a correct integral expression involving $9-(x-3)^2$ **1 Mark:** Identifies $v = +\sqrt{6x - x^2}$

	$v^2 = 6x - x^2$ $v = \pm\sqrt{6x - x^2}$ $x = 3, v = 3 \Rightarrow v = +\sqrt{6x - x^2}$ $\frac{dx}{dt} = \sqrt{6x - x^2}$ $\frac{dx}{\sqrt{6x - x^2}} = dt$ $\int \frac{1}{\sqrt{9 - (x-3)^2}}\,dx = \int dt$ $\sin^{-1}\left(\frac{x-3}{3}\right) = t + C$ $t = 0, x = 3 \Rightarrow C = \sin^{-1}0 = 0$ $\therefore \quad t = \sin^{-1}\left(\frac{x-3}{3}\right)$ $\therefore \quad x = 3\sin t + 3$	*14 students got full marks for Q15a*
15bi	$P(z) = z^4 - 10z^3 + cz^2 + dz + 169$ Coefficients of $P(z)$ are real $\therefore$ complex zeroes occur in conjugate pairs $\therefore$ other two zeroes are $\overline{\alpha} = a - ib$ and $\overline{\beta} = b - ia$	**1 Mark:** Correct answer
15bii	Sum of zeroes $= -\left(-\frac{10}{1}\right) = 10$ $a + ib + a - ib + b + ia + b - ia = 10$ $2a + 2b = 10$ $a + b = 5 \quad (1)$ Product of zeroes $= \frac{169}{1} = 169$ $\therefore \quad (\alpha\overline{\alpha})(\beta\overline{\beta}) = 169$ $\|\alpha\|^2\|\beta\|^2 = 169$ $\|\alpha\|\|\beta\| = 13$ $\sqrt{a^2 + b^2}\sqrt{b^2 + a^2} = 13$ $a^2 + b^2 = 13$ $b = 5 - a \Rightarrow a^2 + (5-a)^2 = 13$ $a^2 + 25 - 10a + a^2 = 13$ $2a^2 - 10a + 12 = 0$ $a^2 - 5a + 6 = 0$ $(a-2)(a-3) = 0$ $a = 2 \text{ or } 3$ $\therefore \quad b = 3 \text{ or } 2$	**2 Marks:** Correct answer **1 Mark:** Finds the sum or product of the zeros *17 students got full marks for Q15b*

15c	$$\left(2-\frac{1}{n}\right)^n > n\ ,\quad n \geq 2$$ $$S(2):\ LHS = \left(2-\frac{1}{2}\right)^2 = \frac{9}{4} \geq 2$$ $$RHS = 2$$ LHS>RHS $S(2)$ is true $S(k)$: $$\left(2-\frac{1}{k}\right)^k > k\ ,\ k \in \mathbb{Z}^+\ ,k \geq 2$$ $$S(k+1):RTP\ \left(2-\frac{1}{k+1}\right)^{k+1} > k+1$$ $$LHS - RHS = \left(2-\frac{1}{k+1}\right)^{k+1} - (k+1)$$ $> \left(2-\frac{1}{k}\right)^{k+1} - (k+1)$, since $k > 0$ $= \left(2-\frac{1}{k}\right)^k\left(2-\frac{1}{k}\right) - (k+1)$ $> k\left(2-\frac{1}{k}\right) - (k+1)$ by $S(k)$ $> 2k - 1 - k - 1$ $= k - 2$ ≥ 0 since $k \geq 2$ $\therefore$ $LHS - RHS > 0 \Rightarrow$ LHS > RHS $\therefore$ $\left(2-\frac{1}{n}\right)^n > n\ ,\ n \geq 2$ by induction	**3 Marks:** Correct answer **2 Marks:** Proves *S*(2), correctly states *S*(k) and *S*(k+1), and forms a correct expression equivalent to LHS-RHS **1 Mark:** Proves *S*(2) and correctly states *S*(k) and *S*(k+1) ***10 students answered correctly***
16ai	Since $OCBA$ is a rectangle, $\overrightarrow{OA}.\overrightarrow{OC} = 0$ $6 - 12 + 3a = 0$ $\therefore a = 2$	**1 Mark:** Correct answer. ***Mostly done well***
16aii	Since $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OG}$, their dot product is equal to zero. $\overrightarrow{OA}.\overrightarrow{OG} = 0$ $-6 - 12y + 3z = 0$ And since $\overrightarrow{OG}$ is perpendicular to $\overrightarrow{OC}$, their dot product is also equal to zero. $\overrightarrow{OG}.\overrightarrow{OC} = 0$ $-4 + y + 2z = 0$ Solving simultaneously gives $y = 0\ , z = 2$	**3 Marks:** Correct answer. **2 Marks:** Uses the dot product to set up two equations and attempts to solve simultaneously **1 Mark:** Finds one correct equation from dot product. ***Mostly done well***

16aiii	$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{OC} + \overrightarrow{OG} = \begin{pmatrix} 3 \\ -11 \\ 7 \end{pmatrix}$ Hence, point E is 7 units above the $x - y$ plane.	**2 Marks:** Correct answer. **1 Mark:** Finds OE ***Mostly done well*** ***17 students got full marks for Q16a.***
16b	$\frac{x^2 - 7x + 4}{(x+1)(x-1)^2} \equiv \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ $x^2 - 7x + 4 \equiv A(x-1)^2 + B(x+1)(x-1) + C(x+1)$ $x = 1 \Rightarrow -2 = 2C \Rightarrow C = -1$ $x = -1 \Rightarrow 12 = 4A \Rightarrow A = 3$ $x = 0 \Rightarrow 4 = 3 + B(1)(-1) + (-1)$ $\therefore \quad 4 = 2 - B \Rightarrow B = -2$ $\therefore \quad I = \int \left[\frac{3}{x+1} - \frac{2}{x-1} - \frac{1}{(x-1)^2} \right] dx$ $I = 3\ln\|x+1\| - 2\ln\|x-1\| + (x-1)^{-1} + K$ **<u>Alternative (but more difficult) method:</u>**	**4 Marks:** Correct answer. **3 Marks:** Finds correct values for *A, B* and *C* and states correct partial fraction decomposition of integrand. OR Finds an expression for *I* consistent with incorrect values for *A,B* and *C*. **2 Marks:** Finds correct values for *A, B, C* **1 Mark:** Forms a correct partial fraction decomposition and attempts to find values for *A, B, C*. ***15 students answered correctly.*** ***Most students didn't choose the easiest partial fraction decomposition.***

	$\dfrac{x^2 - 7x + 4}{(x+1)(x-1)^2} \equiv \dfrac{A}{x+1} + \dfrac{Bx + C}{(x-1)^2}$ $x^2 - 7x + 4 \equiv A(x-1)^2 + (Bx + C)(x+1)$ $x = -1 \Rightarrow 4A = 12 \Rightarrow A = 3$ $x^2 - 7x + 4 \equiv 3(x-1)^2 + (Bx + C)(x+1)$ $x = 0 \Rightarrow 4 = 3 + C \Rightarrow C = 1$ $x^2 - 7x + 4 \equiv 3(x-1)^2 + (Bx + 1)(x+1)$ $x = 1 \Rightarrow -2 = (B+1)(2) \Rightarrow B = -2$ $\therefore \quad x^2 - 7x + 4 \equiv \dfrac{3}{x+1} + \dfrac{-2x+1}{(x-1)^2}$ $\therefore \quad I = \displaystyle\int \frac{3}{x+1}\,dx + \int \frac{-2x+1}{(x-1)^2}\,dx$ $= 3\ln\lvert x+1\rvert + \displaystyle\int \frac{-2x+2}{(x-1)^2}dx - \int \frac{1}{(x-1)^2}\,dx$ $= 3\ln\lvert x+1\rvert - 2\displaystyle\int \frac{x-1}{(x-1)^2}dx - \int (x-1)^{-2}\,dx$ $= 3\ln\lvert x+1\rvert - 2\displaystyle\int \frac{1}{x-1}dx + (x-1)^{-1}$ $= 3\ln\lvert x+1\rvert - 2\ln\lvert x-1\rvert + \dfrac{1}{x-1} + C$	
16ci	$\underset{\sim}{\mathbf{a}} = a_1\,\underset{\sim}{\mathbf{i}} + a_2\,\underset{\sim}{\mathbf{j}} + a_3\,\underset{\sim}{\mathbf{k}}$ $\underset{\sim}{\mathbf{b}} = b_1\,\underset{\sim}{\mathbf{i}} + b_2\,\underset{\sim}{\mathbf{j}} + b_3\,\underset{\sim}{\mathbf{k}}$ $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} = a_1 b_1 + a_2 b_2 + a_3 b_3$ $\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} = \lvert\underset{\sim}{\mathbf{a}}\rvert\lvert\underset{\sim}{\mathbf{b}}\rvert\cos\theta$ Since $-1 \le \cos\theta \le 1$, $-\lvert\underset{\sim}{\mathbf{a}}\rvert\lvert\underset{\sim}{\mathbf{b}}\rvert \le \underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}} \le \lvert\underset{\sim}{\mathbf{a}}\rvert\lvert\underset{\sim}{\mathbf{b}}\rvert$ $\therefore \quad \lvert\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}}\rvert \le \lvert\underset{\sim}{\mathbf{a}}\rvert\lvert\underset{\sim}{\mathbf{b}}\rvert$ $(\underset{\sim}{\mathbf{a}} \cdot \underset{\sim}{\mathbf{b}})^2 \le \lvert\underset{\sim}{\mathbf{a}}\rvert^2\lvert\underset{\sim}{\mathbf{b}}\rvert^2$ $\therefore (a_1 b_1 + a_2 b_2 + a_3 b_3)^2 \le (a_1 + a_2 + a_3)^2(b_1 + b_2 + b_3)^2$	**2 Marks:** Correct answer. **1 Mark:** Obtains dot product and attempts to use $\lvert \cos\theta \rvert \le 1$. *This part was done reasonably well.*

16cii	Let $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} x \\ 2y \\ z \end{pmatrix}$ and $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \\ 1 \end{pmatrix}$ From (i), $\left(x^2 + (2y)^2 + z^2\right)\left(1^2 + \left(\frac{1}{2}\right)^2 + 1^2\right) \geq \left(x \times 1 + 2y \times \frac{1}{2} + z \times 1\right)$ $(x^2 + 4y^2 + z^2)\left(\frac{9}{4}\right) \geq (x + y + z)^2$ $(x^2 + 4y^2 + z^2)\left(\frac{9}{4}\right) \geq 3^2$ $(x^2 + 4y^2 + z^2) \geq \frac{4}{9} \times 9$ $(x^2 + 4y^2 + z^2) \geq 4$ $\therefore$ min value is 4	**3 Marks:** Correct answer. **2 Marks:** Correct values for a_i and b_i components and attempts to use the result from part (i). **1 Mark:** Correct values for either a_i or b_i components. ***4 students got full marks for Q16c. Most students were unsure how to start with part ii.***